



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\therefore AO \cdot BO \cdot CO = abc = \frac{\Delta xyz}{p^2} = 4Rr^2.$$

$$\therefore 4Rr = abc/r, \text{ and } (a^2r + abc)/r = yz, (b^2r + abc)/r = xz, (c^2r + abc)/r = xy.$$

$$\begin{aligned} \therefore x(a^2r + abc)/r &= y(b^2r + abc)/r = z(c^2r + abc)/r \\ &= \sqrt{[(a^2r + abc)(b^2r + abc)(c^2r + abc)/r^3]}. \end{aligned}$$

$$\text{Also } a^2x + b^2y + c^2z = xyz \left(\frac{p-x}{p} + \frac{p-y}{p} + \frac{p-z}{p} \right) = xyz = 1.$$

$$\therefore \frac{a^2}{yz} + \frac{b^2}{xz} + \frac{c^2}{xy} = 1, \text{ or } \frac{ar}{ar+bc} + \frac{br}{br+ac} + \frac{cr}{cr+ab}.$$

$$\therefore 2abc r^2 + (a^2b^2 + a^2c^2 + b^2c^2)r^2 = a^2b^2c^2.$$

$$\therefore r^3 + Ar^2 = B. \text{ Let } r = S - \frac{1}{3}A.$$

$$\therefore S^3 - \frac{1}{3}A^2S = B - 2A^3/27.$$

$$\therefore S = \left(\frac{1}{2}B - \frac{1}{27}A^3 + \sqrt{\frac{B^2}{4} - \frac{A^3B}{27}} \right)^{\frac{1}{3}} + \left(\frac{1}{2}B - \frac{1}{27}A^3 - \sqrt{\frac{B^2}{4} - \frac{A^3B}{27}} \right)^{\frac{1}{3}}.$$

This determines r and therefore x, y, z , the sides of the triangle.

Otherwise, draw AO and produce AO to O_1 so that $OO_1 = bc/r$. Upon OO_1 as diameter describe a circle. With O as center and b as a radius describe an arc cutting the circle in B . Similarly, with O as center and c as radius, draw an arc cutting the circle in C . Join BC, AC, AB , then ABC is the triangle required. For O_1 is the ex-center opposite A by construction as follows:

$$AO \cdot AO_1 = yz = (AO^2r + AO \cdot BO \cdot CO)/r.$$

$$\therefore AO_1 = (AO^2r + BO \cdot CO)/r.$$

The solution published in the last issue contained an error in the fourth line, and this vitiated the whole solution. Ed.

163. Proposed by J. C. NAGLE, Professor of Civil Engineering in the Agricultural and Mechanical College of Texas, College Station, Tex.

Given the equal sides of an isosceles triangle and the radius of the inscribed circle to solve the triangle. As a numerical example let the known sides be 27 and the radius of the inscribed circle 3.5. The problem occurred in connection with some mill work and the exterior angles of the triangle were required in order to make patterns for iron braces.

I. Solution by A. H. HOLMES, Professor of Mathematics, Bowdoin College, Brunswick, Me.

Let ABC be the triangle, O the center of the inscribed circle, and OD the perpendicular from O to the side AB , its radius; let $\angle BOD = \theta$ and $\angle AOD = \phi$. Then we have $\tan \theta + \tan \phi = \frac{5}{4}$. But $\phi + 2\theta = 180^\circ$.

$$\therefore \tan \phi = -\tan 2\theta = -\frac{2\tan \theta}{1 - \tan^2 \theta}.$$

$$\therefore \tan^3 \theta - \frac{5}{4}\tan^2 \theta + \tan \theta + \frac{5}{4} = 0.$$

From this equation we find $\tan \theta = 1.16425$.

$$\therefore \angle ABC = \angle ACB = 81^\circ 19' 14'', \text{ and } \angle BAC = 17^\circ 21' 32''.$$

We now readily find $BC = 8.149 +$.

II. Solution by H. C. WHITAKER, C. E., Ph. D., Manual Training School, Philadelphia, Pa.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and L. C. WALKER, A. M., Petaluma High School, Petaluma, Cal.

$$r = (s - a)\tan \frac{1}{2}A = \frac{1}{2}(b + c - a)\sqrt{\frac{(a + c - b)(a + b - c)}{(a + b + c)(b + c - a)}}.$$

$$\text{But } b = c. \quad \therefore r = a\sqrt{\frac{2b - a}{2b + a}}.$$

$$\therefore a^3 - 2a^2b + 4ar^2 + 8br^2 = 0.$$

This cubic can be solved, and gives the values of the unknown sides.

Substituting values, we get $a^3 - 54a^2 + 49a + 2646 = 0$.

$\therefore a = 8.1498, 52.0838, \text{ or } -6.2336$. The first and second values are both admissible. $\cos B = a/2b = .15092$ or $.96451$.

$$\therefore B = 81^\circ 19', \text{ or } 15^\circ 18'.$$

Also solved by J. SCHEFFER.

We received a solution of problem 156 from Prof. Henry Heaton too late for credit in the number in which the solution of the problem was published. Professor Heaton gave construction also when the point does not lie between the two parallels. As the construction is similar to the case when the point lies between the two parallels we will not publish the construction.

CALCULUS.

REMARKS ON THE SOLUTION OF PROBLEM 123.

BY WM. E. HEAL, MARION, IND.

A remarkable error occurs in the published solution of Problem 123, in writing $1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \dots = \frac{\pi^3}{35}$.

The summation referred to by Professor Echols and the Editor is incorrectly quoted. The true formula is

$$\tau_{2m+1} = \frac{E_m \pi^{2m+1}}{2^{2(m+1)}(2m)!} = 1 - \frac{1}{3^{2m+1}} + \frac{1}{5^{2m+1}} - \dots$$

See Chrystal's *Algebra*, Part II, page 342. For $m=1$ this becomes, since $E_1=1$,